Math Challenge 2008 – Multiple Choice Questions

1. If the area of a circle is given by \( A = \pi r^2 \) and the circumference is given by \( C = 2\pi r \), then a correct formula for area in terms of circumference is
   a. \( A = \frac{C}{4\pi} \)  
   b. \( A = \frac{C^2}{2\pi} \)  
   c. \( A = \frac{C^2}{4\pi} \)  
   d. \( A = \frac{C^2}{4} \)

2. If \( p = x - 1 \) and \( q = x + 1 \) then \( \sqrt{q^2 - p^2} = \)
   a. \( 2\sqrt{x} \)  
   b. 0  
   c. \( \sqrt{2} \)  
   d. \( 4x \)

3. Square \( S_1 \) has a diagonal of length \( d \). Square \( S_2 \) has a diagonal of length \( 2d \). When the area of \( S_2 \) is divided by the area of \( S_1 \), the result is
   a. \( \sqrt{2} \)  
   b. 2  
   c. \( 2\sqrt{2} \)  
   d. 4

4. Here are five numbers: 111; 11,111; 111,111; 1,111,111; 111,111,111,111. Exactly how many of these numbers are evenly divisible by 3?
   a. 1  
   b. 2  
   c. 3  
   d. 4

5. What is \( x \)?
   ![Diagram](image)
   a. \( \sqrt{15} \)  
   b. \( \frac{\sqrt{75}}{3} \)  
   c. 5  
   d. \( \frac{5}{\sqrt{3}} \)
6. 5% of $\frac{1}{2}$ of $\frac{1}{4}$ is
   a. 0.625  b. 0.0625  c. 0.00625  d. 2.5

7. When a fair coin is flipped, the probability that it comes up heads is $\frac{1}{2}$. If this coin is flipped eight times, then the probability that it comes up heads all eight times is
   a. $\frac{1}{16}$  b. 0  c. 4  d. $\frac{1}{256}$

8. Assume $x \leq y \leq z$. If the mean of $x$, $y$, and $z$ is 10 and the median of $x$, $y$, and $z$ is 5, then $x+z$ is
   a. 5  b. 10  c. 25  d. Impossible to determine

9. Find $x$.

10. Lyon Street goes downhill as indicated in the figure. If you were to drive $\frac{1}{10}$ of a mile on Lyon Street, how much would your altitude change? (5280 feet = 1 mile)

   a. 52.8 ft  b. 528 ft  c. $528\sin(10^\circ)$ ft  d. $528\tan(10^\circ)$ ft
11. How many different three-letter “words” can be created using the letters in MARCH if no letter can be used more than once? (A “word” does not have to be an actual meaningful word in any language).

a. 60  
b. 10  
c. 120  
d. 6

12. In a certain area of Kent county the number of home foreclosures increased from 1 in 2004 to 19 in 2007. What is the percentage increase in home foreclosures?

a. 19%  
b. 1900%  
c. 18%  
d. 1800%

13. If the larger circle has radius \( A \) and the smaller circle has radius \( B \) then the area of the shaded region is

a. \( \pi \left( A^2 - B^2 \right) \)  
b. \( \pi \left( A^2 + B^2 \right) \)  
c. \( \pi \left( A - B \right)^2 \)  
d. \( \pi A^2 - B \)

14. \( \left( 2^{x-y} \right) \div \left( 2^{x+y} \right) = \)

a. \( 4^{x-y} \)  
b. \( 2^{-2y} \)  
c. 1  
d. \( 2^{-2x} \)

15. Let \( \# \) be an operation defined by \( x \# y = x^2 - y^2 \) and let \( * \) be an operation defined by \( x \ast y = \left( \frac{x}{y} \right)^{\frac{1}{2}} \). Find \( 4 \ast (5 \# 3) \).

a. 64  
b. \( \frac{1}{4} \)  
c. \( \frac{1}{2} \)  
d. 60
16. The slope of the line shown is

![Graph of a line with points (-2, -1) and (0, 0)]

a. $\frac{1}{2}$  
   b. $-\frac{1}{2}$  
   c. 2  
   d. -2

17. The mean of all prime numbers between 70 and 100 is

a. $83 \frac{2}{7}$  
   b. 82  
   c. 81  
   d. 81.8

18. If the figure is a regular octagon then the measure of angle $\theta$ is

![Regular octagon]

a. 135°  
   b. 150°  
   c. 120°  
   d. $137 \frac{1}{2}$°

19. Assume that the room in which you are taking this test is a rectangular box measuring 28 feet by 30 feet by 11 feet. If a shoebox measures 4 $\frac{1}{2}$ inches by 6 inches by 13 inches, approximately how many shoeboxes could fit in this room?

a. 45,489  
   b. 316  
   c. 3,791  
   d. 423

20. A car travels west on I-96 at 70 miles per hour. The driver notices that it takes one minute to pass 125 of the thin white rectangles that separate the two westbound lanes (see figure). Find the approximate length of each white rectangle; that is, find $x$. (5280 feet = 1 mile)

![Diagram of white rectangles]

a. 49.3 feet  
   b. 16.5 feet  
   c. 14.1 feet  
   d. 42.2 feet
21. The sum of all prime factors of 2008 is
   a. 266  b. 258  c. 257  d. 40

22. At 6:35 p.m. the angle between the minute hand and the hour hand of a clock is
   a. 12.5°  b. 15°  c. 17.5°  d. 10°

23. $2 \times 3 \times 4 \div 4 \times 3 \times 2 =$
   a. 1  b. 0  c. 36  d. 18

24. If $x$ and $y$ are real numbers the equation $(x+y)^2 = x^2 + y^2$ is true under what conditions?
   a. Always  b. Never  c. Only if $x = y$  d. Only if at least one of $x$ or $y$ is 0

25. $(3^{-1} - 2^{-1})^{-1} =$
   a. 1  b. $-1$  c. 6  d. $-6$

26. In 2006 Sara’s salary was D dollars. In 2007 she got a 10% raise in pay. Then in 2008 she got a 10% cut in pay. What was her final salary, in dollars?
   a. 0.99D  b. D  c. 0.90D  d. 1.01D

27. If the area of a square is 5, then its perimeter is
   a. $\sqrt{5}$  b. 10  c. $4\sqrt{5}$  d. $\frac{5}{4}$
28. The set of \( x \) for which \( -\sqrt{-x^2} \) is a real number is

a. All real numbers  
b. The empty set  
c. \( \{0\} \)  
d. \( \{-1, 1\} \)

29. If \( f(x) = -x^4 \), then \( f(-2) = \)

a. \(-16\)  
b. \(16\)  
c. \(-8\)  
d. \(8\)

30. In the figure \( D, E, \) and \( F \) are the midpoints of segments \( AB, BC \) and \( CA \) respectively. Also \( X, Y, \) and \( Z \) are the midpoints of segments \( DF, DE \) and \( EF \) respectively. If the length of \( AB = BC = CA = 8 \), then the area of triangle \( XYZ \) is

![Diagram of triangle ABC with midpoints marked]

a. \(4\sqrt{3}\)  
b. \(2\)  
c. \(4\)  
d. \(\sqrt{3}\)

31. If a rectangular block of ice measuring 4.00 inches by 5.00 inches by 6.00 inches weighs 4.17 pounds, then a rectangular block of ice measuring 10.00 inches by 12.50 inches by 15.00 inches would weigh

a. 10.43 pounds  
b. 26.06 pounds  
c. 65.16 pounds  
d. \(20\pi\) pounds
32. On a day when $1$ Canadian = $1.09$ U.S., gas in Toronto, Ontario, Canada cost $0.78$ Canadian per liter. What would be the equivalent cost in U.S. dollars per gallon?

(1 liter ≈ 1.06 quarts, 4 quarts = 1 gallon)

a. $3.03$  
   b. $2.70$  
   c. $3.60$  
   d. $3.21$

33. Let $c > 0$. The equation $x^2 - x - c = 0$ has exactly

a. One real root  
   b. Two real roots  
   c. One complex root  
   d. Two non-real complex roots

34. Suppose $w, x, y,$ and $z$ are positive real numbers. If $w \cdot x = 2$, $x \cdot y = 3$ and $y \cdot z = 5$, then $w \cdot z =$

a. $60$  
   b. $\frac{10}{3}$  
   c. $\frac{3}{10}$  
   d. $15$

35. In the figure the following information is known: length of $AB = 10$, length of $BC = 15$, length of $CA = 17$. Segment $BQ$ bisects angle $ABC$ and segment $CQ$ bisects angle $ACB$. Finally, $PR$ is parallel to $BC$. What is the perimeter of triangle $APR$?

!! Image !!

a. 21  
   b. 24  
   c. 27  
   d. 30
Math Challenge 2008 – Multiple Choice Answer Key

1. c
2. a
3. d
4. c
5. a
6. c
7. d
8. c
9. b
10. c
11. a
12. d
13. a
14. b
15. c
16. b
17. b
18. a
19. a
20. b
21. c
22. a
23. c
24. d
25. d
26. a
27. c
28. c
29. a
30. d
31. c
32. d
33. b
34. b
35. c